Journal of Science Education
Volume: III, Issue I, 2021, 63-80

Teaching Direct and Inverse proportion to 5th Grade Students through Pictures and Real-Life Problems, Leading Towards Autonomous Learning

Tabassum Saleem[*]
Surayya Mukhtar[**]
Shamsa Aziz[***]

Abstract

Mathematical concepts of ratio and proportions are primarily included in the mathematics syllabus at the 5th-grade level. The concept of proportion is quite tricky for students. Most of the questions based on the concept of proportion are in the form of word problems. The present study is experimental, and it aimed to find an easier way of explaining the concept of proportion in mathematics for 5th-grade students. There was a total of 68 students, and two groups (controlled and experimental) were made with 34 students in each group. The duration of the experiment was two weeks, and the same teacher taught both groups. The experimental group was acquainted with pictures and real-life problems to explain the concept of proportion. In contrast, the controlled group was taught without these teaching aids. At the end of the experiment, a post-test was administered to both groups. Data were analyzed through an independent sample t-test. The findings of the study showed that the experimental group showed better results. Some techniques were used to analyze the autonomous learning of the students. Again, data analysis showed that experimental groups showed better performance. Based on the study's findings, it was recommended that complex mathematical concepts be taught using proper audio-visual aids. Teachers need to improve their teaching methodologies, especially for teaching the concepts of ratio and proportion.

Keywords: direct proportion, inverse proportion, autonomous learning.

[*] Department of Education, International Islamic University, Islamabad, Pakistan.
Email: tabassumsaleem786@yahoo.com
[**] Assistant Professor, Department of Physics, Allama Iqbal Open University, H-8, Islamabad.
Email: Surayya.mukhtar@aiou.edu.pk
[**] Associate Professor, Department of Education, International Islamic University, Islamabad.
Email: shamsa.aziz@iiu.edu.pk

Introduction

Ratio and proportions are the two critical mathematical concepts included in mathematics books, usually at grade 5. The ratio can be defined as "the relation between two or more values or quantities", while proportion can be defined as "the relation between two or more ratios". These concepts will become the base for many complex concepts of physics and mathematics at a higher level. In physics, the concept of direct relation and inverse relation is frequently used. Similarly, in mathematics equations of straight lines, graphs mainly depend upon the relation between certain values. Autonomous learning can be defined as an "approach to learning with which learners hold power or right to regulate and control their learning activities". Learners are in charge of their learning processes with autonomy.

It has been observed that at the primary level, the main problem with these concepts is two-folded. One is the understanding of the statement, as most of the mathematics book questions related to these topics are in the form of word problems. Secondly, the identification of the variables, that is, the variables among which we had to find the direct or inverse relation.

The present study was focused on solving these issues by using real-life problems and pictures, which showed the situations of the word problems in pictorial form.

Objectives of the Study

1. To use new methods (real-life problems and pictures) to explain the concept of direct and inverse proportion at the primary level.
2. To evaluate the effectiveness of these methods at students' learning at the primary level.
3. To find the students' problems in understanding direct and inverse proportion.
4. To evaluate the effectiveness of these methods in enhancing autonomous learning of students at the primary level.

Hypotheses of the Study

H_{01}. There is no significant difference in the mean achievement scores between the students taught with and without using real-life problems and pictures.

H_{02}. There is no significant difference in the Autonomous learning abilities between the students taught with and without using real-life problems and pictures.

Significance of the Study

The present study is significant for teachers as they may get knowledge about students' problems in understanding the concept of direct and inverse proportion at the primary level. It is significant for test developers and teachers that they would become familiar with the use of real-life problems and use of pictures for taking tests for students at the primary level.

Statement of the Problem

Ratio and Proportions are the two essential topics that start mainly at the primary level in schools. These concepts are used in Algebra and higher mathematics at secondary and higher secondary school levels. The problems related to direct and inverse proportion are mainly in the form of word problems in mathematics books.

Students faced two main issues at this level. One is the new concept for them, and the second is that the sums are in the form of word problems. It has been observed that by using Real-life problems and using pictures, these problems can be solved.

Literature Review

It is a useful technique of mathematics education, that the students try to find connections and relations among different mathematical concepts and ideas so that they can use them for future mathematical learning (Mathematics, 2000). To accomplish this goal, teachers should teach mathematics to students on a variety of levels, including fragmented instruction and drill (CCSSI, 2010). So that students can recognize and exploit diverse relationships between mathematical ideas (CCSWT, 2011). The concept of direct and inverse proportion is one of the predominant concepts of the elementary and middle school mathematics curriculum (DeJarnette, Walczak, & González, 2014). It is an essential element for the understanding of algebra and beyond (Lesh , Post , & Behr, 1988).

Students faced many problems while solving direct and inverse proportion sums. The researches have enlisted the students' following six problems while solving the mathematical problems based on the direct and inverse proportion (Irfan, Nusantara, Subanji, & Sisworo, 2018). Other researchers also have the same thinking that students are unable to

recognize proportional (direct and inverse) relationships in solving the problems based upon the concept of direct and inverse relation (Lobato & Ellis, 2010).

1. don't understand the use of variables
2. don't understand the use of formulas
3. don't understand the key phrases on the problem
4. don't understand the difference in ratio, fractional, and division
5. don't understand the word problems
6. don't understand the simplification of division
7. don't interpret proportion relation correctly

The research conducted by Arican(2019) also showed that seventh-grade middle school students' overall weaknesses in all four attributes understood ratios, directly, inversely, and non-proportional relationships. Based on his findings, he further suggested that teachers should expand students' knowledge of ratios, proportions, and proportional relationships to develop a meaningful understanding of these concepts (Arican, 2019).

Lamon in his book "An integration of research", has explained the students' problems in understanding complex mathematical concepts. In the book chapter with the title "Ratio and Proportion: Children cognitive and metacognitive processes," he further explained that ratio is viewed as a complex unit resulting from several compositions (Lamon, 1993).

The problems based upon proportion are solved mainly by the cross-multiplication by taking the unknown value as 'x'. Research conducted in Indonesia showed that about proportion, students have different levels of intuitive understanding. Students who are unable to solve a simple proportional problem (missing value) are also unable to work on the comparison problem, that contains more than one set of numbers. Even those students who can solve a proportional problem may still have difficulties in solving comparison problems (Sumarto, Galen, Zulkardi, & Darmawijoyo, 2014). The results of a study conducted by Irfan, Nusantara, Subanji, & Sisworo (2019) indicated that students experiencing proactive interference caused by failure to coordinate the knowledge they have with the problems faced. Because of that when students are resolving the problem of inverse proportion, they actually use the concept of direct proportion to solve inverse proportion problems Irfan et al. (2019).

Nursa, Hartono & Somakim (2020) investigated that through learning trajectories, students investigated their knowledge of understanding both proportions. He and his coworker discovered that students were able to recognize and understand direct proportion and are able to solve the problem using a Ratio table in a particular context (Nursa, Hartono, &

Somakim, 2020). With the help of the Ratio, the concept of direct and inverse proportion can be clarified. It also helps them to understand and solve the problems. The findings of the study conducted by Fernández, Linares, Dooren, Bock, & Verschaffel (2012) indicated that in order to solve the proportional problems mostly students used additive methods. And this trend is common in primary school students and decreased in secondary school students. This study also showed that while solving proportional problems students made mistakes in converting the additive methods into multiplicative methods (Fernández, Linares, Dooren, Bock, & Verschaffel, 2012). Students faced more difficulties in solving problems related to inverse proportion as compared to the problems involving the concept of direct proportion (Arican, 2019).

Saifullah & Qohar (2020) used "Simulation Based Sticker Book on Direct and Inverse Proportion" and found that it is helpful in explaining the concept of direct and inverse proportion to the students (Saifullah & Qohar, 2020). Autonomous learning can be defined as an approach to learning with which learners hold power or the right to regulate and control their learning activities. Learners are in charge of their own learning processes with autonomy. Autonomous learning is also called self-directed learning.

Autonomous learning is learning activities that take place more driven by their own will, choices, and responsibilities. Students need to have autonomous learning so that they are responsible for regulating and disciplining themselves and developing learning skills of their own volition. This is in line with Suhendri s' work that learning autonomous is an essential element in learning mathematics (Suhendri, 2011). Students can improve their understanding of mathematics. Autonomous learning allowed the students to be agents (enabling them to be the doers rather than recipients of the learning), which helps sustain their motivation (Harmer, 2007).

Studies on learning autonomy mainly were conducted on adult learners and hardly found on those of young learners. Many people often underestimate the potential for self-regulation in children, seeing them frequently as too young to be self-controlled (Padmadewi, 2016). Students' autonomous learning can be evaluated using different techniques. Padmadewi has mentioned some techniques that include classroom observation, response to various tasks, etc.

Methodology

Research Design

A Pretest-Posttest Design was used to compare students' math achievement when taught using real-life problems and pictures and conducted without the use of real-life examples and pictures.

The Pretest-Posttest Design was as follows:

O1 X1 O2
O3 X2 O4

Where

O1 and O3 = pretests
O2 and O4 =posttest
X1 = treatment
X2 = without treatment

Population

A total of 17.377 million students attend Pakistan's primary schooling system. Of these, 11.463 million (66%) attend public schools, while 5.913 million (34%), attend private schools (Amin, 2013). Private schools in Rawalpindi enroll about 5.913 million students at the primary level (Amin, 2013). All 5th-grade students in district Rawalpindi's private schools were included in the study.

Sample

This research study was conducted on grade five students from a school in Rawalpindi. There was a total of 64 students (males and females), who were divided into two sections A and B.

A sample of 32 students was taken from each group: 15 girls and 17 boys were part of Section A's controlled group, while 13 girls and 19 boys were part of Section B's experimental group.

In this study, only two sections of grade five were taken. The pretest results indicate almost the same number of students in both groups (results from the control and experimental groups). The controlled group and experimental group had the almost same ratio of average, above average, and below-average students.

Teaching Conditions

Both groups (controlled and experimental) received the same instruction from the same teacher. For both groups, the period was forty minutes long. The study lasted for two weeks.

Details of the Experiment

The controlled group was taught by using the usual chalk and board method. While the experimental group was taught by using different activities. As it was already stated that most of the problems related to direct and inverse proportion are in the form of word problems. Some questions were given below.

Some questions from grade 5 books.

Q-1 A worker is paid Rs 360 for three days. What should he be paid for 20 days?

Q-2 15 workers do work in 35 days. In how many days 21 workers will do the same work?

Before starting the exercise, it was essential to explain the concept of direct proportion and inverse proportion. For that purpose, in the experimental group following activities were conducted.

ACTIVITY 1

- Any four students arrange things, and time was calculated
- Then six students arrange the same things, and again time was calculated
- In this way concept of inverse proportion was explained.

ACTIVITY 2

- Six students had to arrange the class
- Three students had to arrange the class

While in the controlled group, the definitions of direct and inverse proportion were written on the board and explained verbally.

In the experimental group, pictures used while in a controlled group were explained on the board and verbally explained. AS an example, one question is described here.

Q-1 A worker is paid Rs 360 for three days. What should he be paid for 20 days?

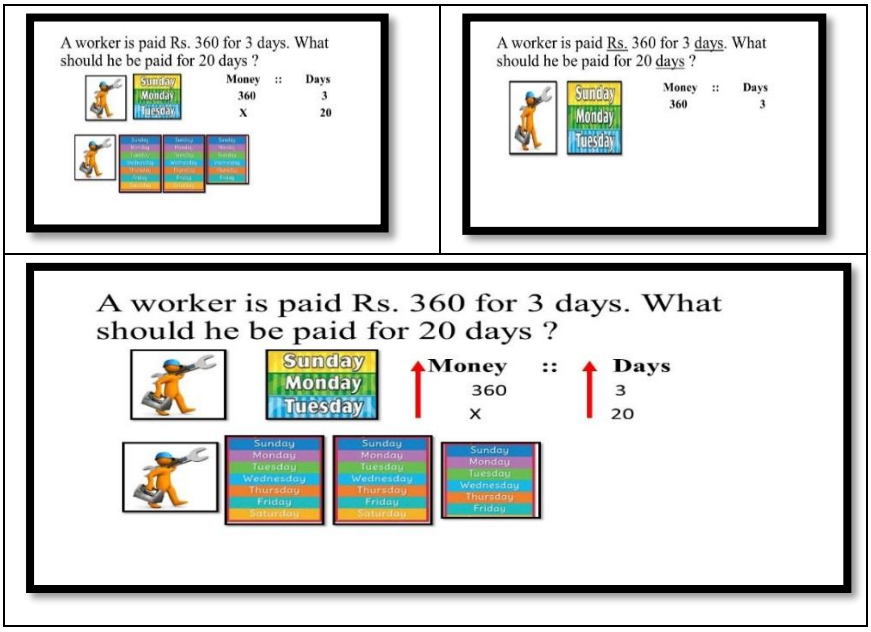
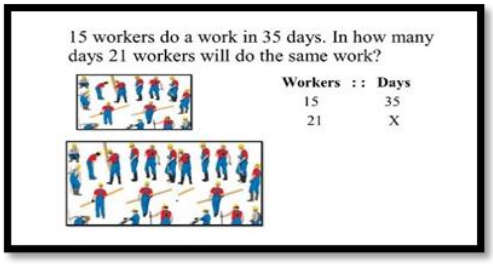


Figure 1. Q-1 with the use of pictures

Using these pictures was straightforward to explain how the days and salaries were directly proportioned to each other.

For each exercise question, pictures were provided to the students and explained the concept of direct proportion and inverse proportion. The question below was the question related to the inverse proportion.

Q-2: 15 workers do work in 35 days. In how many days 21 workers will do the same work?



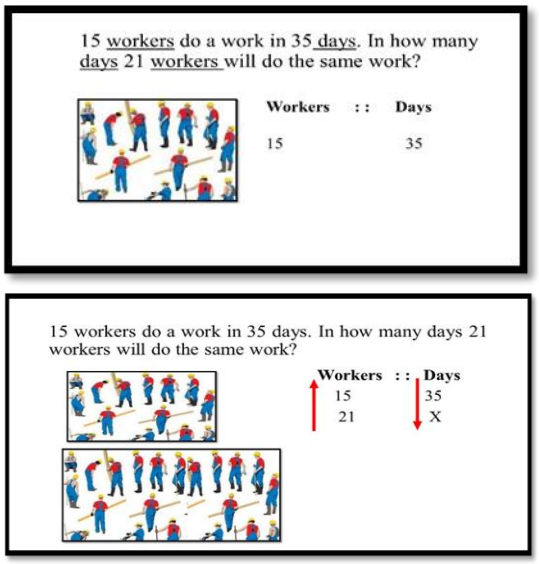


Figure 2. Q-2 with the use of pictures

It was observed that students took a lot of interest while solving these sums.

Autonomous Learning

To enhance autonomous learning, many other activities were done in an experimental group class.

1. Different pictures were shown to them to find the direct and inverse proportion.

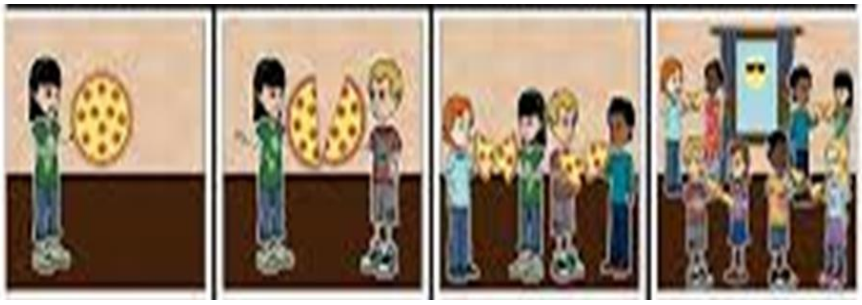


Figure 3. Pictures used to explain the concept of direct and inverse proportion

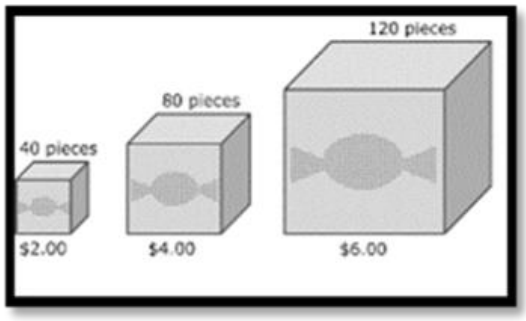


Figure 4. Picture used to find autonomous learning in students during post-test.

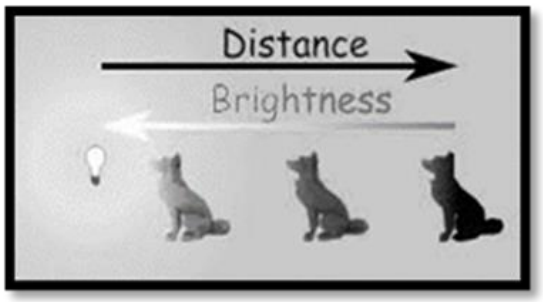


Figure 5. Picture used to find autonomous learning in students during posttest.

2. Students had to bring from their homes at least one pair of things which are either directly proportional or inversely proportional, e.g. number of pencils and size of pencil box etc.
3. Students had to draw pictures of a different object to show direct and inverse relations.

While no such activities were done in the controlled group.

After two weeks post-test was taken from both the groups, and the results were calculated.

Details of Post-test

At the end of the experiment, a post-test was taken from the students of both groups.

Post-test contained objective and subjective portions to evaluate the students' understanding about ratio, direct, and inverse proportion. While checking the autonomous learning of the students' different tasks were given to the students. Each task was given one mark, and then data was analyzed.

Table 1.
Details of Post-test

S. No	Test Items	Marks
1.	Proportion (Direct and inverse)	10
2.	Autonomous Learning (5 tasks)	5

Some samples form students' post-test results.

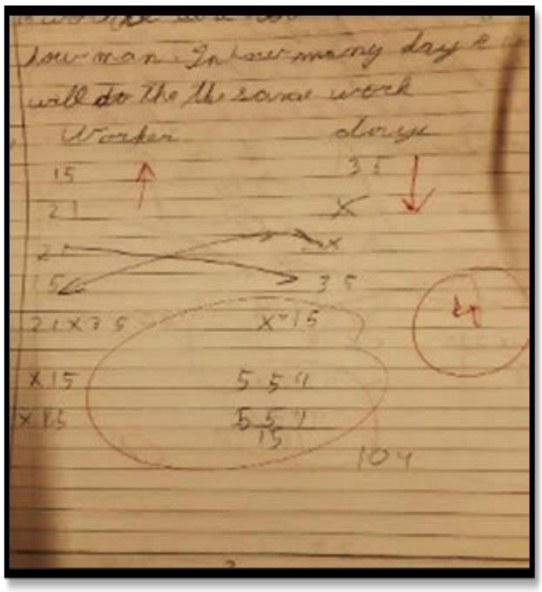


Figure 6. Post-test work of controlled group student.

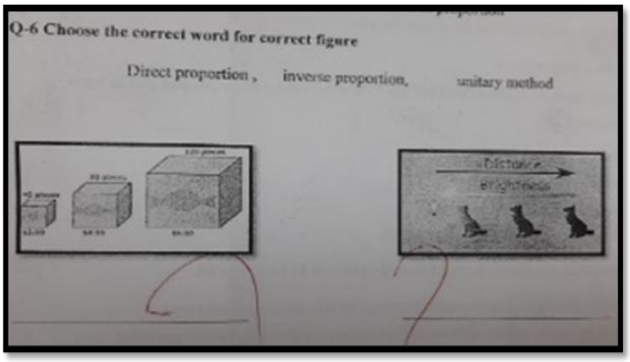


Figure 7. Post-test result of controlled group student.

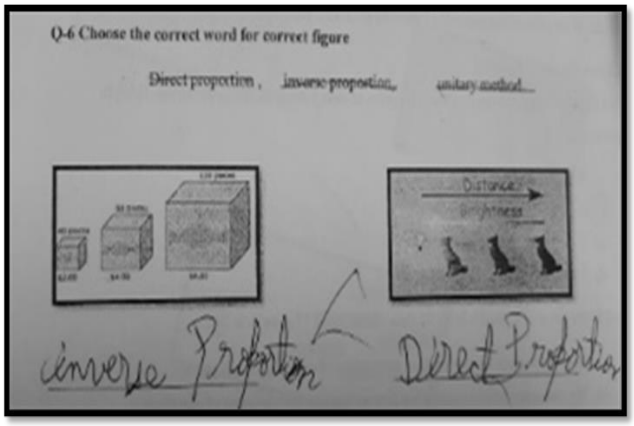


Figure 8. Post-test result of Controlled group students.

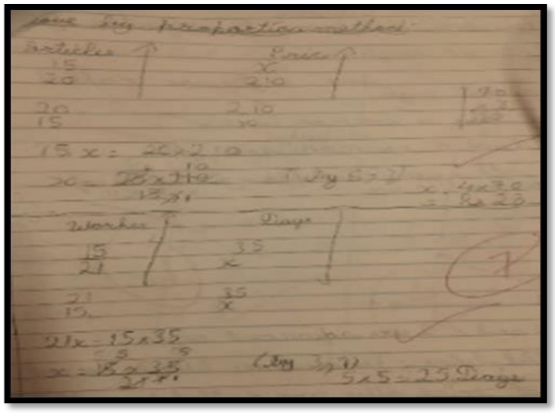


Figure 9. Post-test result of experimental group student

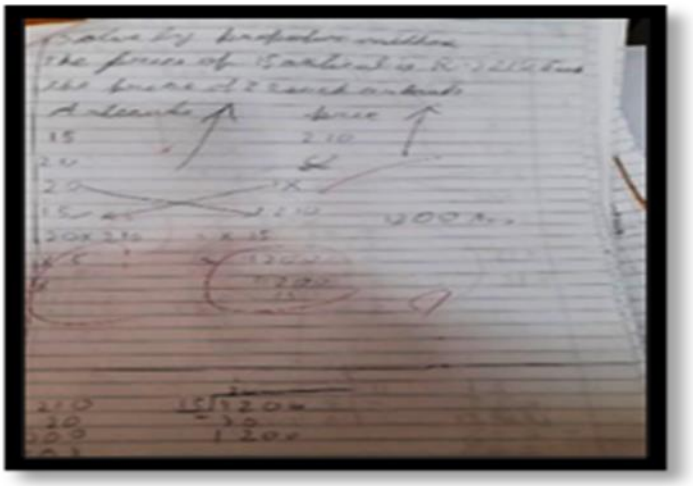


Figure 10. Post test result of experimental group student

Data Collection

At the end of the experiment, a post-test was taken.

Data Analysis

Data were analyzed through a computer by using SPSS software, and hypotheses were tested through mean and independent sample t-test. The results were then interpreted.

Table 2.
Posttest Results for Control and Experimental Groups

S. No	Group	N	Mean	t	df	p-value
1.	Controlled	32	4.96			
2.	Experimental	32	9.06	11.827	62	<0.001

The p-value for the two-tailed test is< 0.01, that value is less than 0.05 at a 5% level of significance, so there is a significant difference between the achievement of both groups (controlled and experimental). Hence the null hypothesis was rejected and the alternative hypothesis is "there is a significant difference between the students' achievement in mathematics of both groups (controlled and experimental) when taught through the use of real-life problems and pictures and without the use of these methods".

To evaluate the autonomous learning abilities, different tasks were given to the students of both groups. While taking post-test of the students, some questions were also asked to check the students' autonomous learning.

The marks of these questions were separately calculated, and an independent-sample t-test was applied. The results were as followed.

Table 3.
Details of Scores for Autonomous Learning

S. No	Group	N	Mean	t	df	p-value
1.	Controlled	32	2.65	7.061	64	<0.001
2.	Experimental	32	4.40			

Considering the p-value was less than 0.05, it was rejected, there was no significant difference between the students' autonomous learning abilities of the controlled group and experimental group. There was, however, a substantial difference in the students' autonomous learning abilities between the two groups.

Data were further analyzed to find the other difficulties faced by students in solving the problems related to the concepts of direct and inverse proportion.

Following five types of difficulties in understanding the concepts of direct and inverse proportion were enlisted while analyzing the posttest questions solved by the students.

Table 4.
Details of the Difficulties in Solving the Sums Based on the Direct Proportion and Inverse Proportion

S. No	Nature of problems	No. of Controlled group students faced these difficulties.		No. of experimental group students faced these difficulties.	
1.	Identification of variables.	15	47%	4	13%
2.	Arrangement of values	12	38%	5	16%
3.	Identification of unknown value.	15	47%	4	13%
4.	Division and multiplication	16	50%	7	22%
5.	Finding the correct answer in terms of an unknown variable	16	50%	7	22%

A detailed study of the questions solved by the students of both the groups (controlled & experimental) showed that most of the students have difficulties in solving the sums based on the direct proportion and inverse

proportion. These difficulties were enlisted in table 4. Some students in the Experimental group had identified the variables correctly, and the concept of direct and inverse proportion was also evident. Still, they were unable to do the rest of the mathematical work. One such test was shown in figure 10.

The above figure showed the sum done by a student of the experimental group. Although the student had identified the variables correctly and directly, they failed to carry out further mathematical calculations that involved multiplication and division.

Discussions

Ratios and proportions are important mathematical concepts. These concepts are taught to the students at the primary level. The students' post-tests analysis showed that the students in the controlled group mostly had problems identifying the variables and then identifying the direct and inverse relation that is in accordance with the research work done by Lobato. He also showed that students don't understand the uses of variables and have problems understanding the phrases used in questions (Lobato, 2010). Proportion questions are usually in the form of word problems, and in mathematics, the word problem itself is a problem. The students of the experimental group successfully solved the same questions in the post-test. The use of pictures and real-life problems helped them to understand these word problems in a better way. The mean score of the experimental group is shown in table 2. The mean score of the experimental group was 9.06 while that of the controlled group was 4.96, which also explained that the use of pictures and real-life examples helped the students understand the concepts of direct and inverse proportion better.

Similarly, the data shown in Table 4 showed the details of the students' difficulties while solving the sums of direct and inverse proportion. This data also supported the fact that students of the experimental group understood the concepts of direct and inverses proportion, identifying variables, and identifying unknown values much better than controlled group students. Some students 7 out of 32 (22%) made mistakes while doing further mathematical calculations based upon division and multiplication, but that number was less than the control group, which was 16 out of 32 (50%). That also supported the study of Lobato that students don't understand the simplification of division while solving the problems of direct and inverse proportion (Lobato, 2010).

Conclusions

Based on the finding of the study, it was concluded that:

1. Students can improve their learning if the teacher will use real-life examples while teaching some difficult concepts such as ratio and proportion.
2. These concepts could be well understood if pictures showing direct and inverse relations have been shown to the students.
3. It was also observed that students faced problems in further mathematical calculations after understanding the concepts of direct and inverse proportion.
4. If students understand the concepts, clearly it would lead towards autonomous learning.

Recommendations

Based on the results of the present study, it is recommended that:

1. Teachers may use real life examples to explain these and other mathematical concepts during daily classroom teaching.
2. Curriculum developers may include more real-life problems and activities in mathematics texts books for a better understanding of mathematical concepts.
3. Teachers' training programs may include the use of more real-life problems and activities. So, teachers should be trained enough to use these methods in their daily teaching.
4. More research may be done to explore the effects of use of real-life examples for the teaching of other mathematical concepts e.g. area and perimeter, volume of two dimensions and three-dimension shapes etc.
5. It is also recommended that further research may be done on other grade levels. As, the present study was focused on grade five students. More research may be done on grade seven and grade eight students to explore the effect of daily life examples to clarify the mathematical concepts.
6. Further studies may be done to understand the difficulties faced by the students while doing division and multiplication of the sums related to direct and inverse proportion sums.

References

Amin, N. (2013). *Pakistan Education Statistics 2011-12.* Islamabad: The NEMIS Team.

Arican, M. (2019). Facilitating pre-service mathematics teachers' understanding of directly and inversely proportional relationships using hands-on and real-world problems. *International Journal of Research in Education and Science (IJRES), 5(*1), 102-117.

Common Core State Standards Initiative (2010). The common core state standards for mathematics. Washington, D.C.: Author. Retrieved from http://www.corestandards.org/assets/CCSSI_Math%20Standards.pdf.

Common Core Standards Writing Team (2011). Progressions for the Common Core State Standards for Mathematics (draft), 6-7, ratios and proportional relationships. Retrieved from http://math.arizona.edu/~ime/progressions.

DeJarnette, A. F., Walczak , M., & González, G. (2014). Students' concepts and theorems-in-action on a novel task about similarity. *School Science & Mathematics, 114(*8), 405-414.

Fernández, C., Llinares, S., Van Dooren, W., De Bock, D., & Verschaffel, L. (2012). The development of students' use of additive and proportional methods along with primary and secondary school. *European journal of psychology of education*, *27(*3), 421-438.

Harmer, J. (2007). *The Practice of English Language Teaching*. England: Pearson Education Limited.

Irfan, M., Nusantara, T., Subanji, S., & Sisworo, S. (2018). Why Did the Students Make Mistakes in Solving Direct and Inverse Proportion Problem? *International Journal of Insights for Mathematics Teaching (IJOIMT)*, *1*(1), 25-34.

Irfan, M., Nusantara, T., Subanji, & Sisworo. (2019). Direct Proportion or Inverse Proportion? The Occurrence of Student Thinking Interference. *Internation Journal of Scientific & Technology, 8(*7), 587-590.

Lamon, S. J. (1993). Ratio and proportion: Children's cognitive and metacognitive processes Rational numbers. In S. J. Lamon, *An integration of research* (pp. 133-156). *New Jersey: Lawrence Erlbaum Associates*.

Lesh, R., Post, T., & Behr, M. (1988). Number concepts and operations in the middle grades. In H. J, & B. M, *Proportional reasoning* (pp. 93-118). Reston, VA: National Council of Teachers of Mathematics.

Lobato, J. & Ellis, A. (2010). *Developing essential understanding of ratios, proportions, and proportional reasoning for teaching mathematics: Grade 6-8.* Reston VA: National Council of Teachers of Mathematics.

Mathematics, N. C. (2000). *Principles and standards for school mathematics.* Reston VA: National Council of Teachers of Mathematics.

Nursa, N. F, Hartono, Y. & Somakim, A. (2020). On teaching learning for proportion using musi tour context. *National Conference on Mathematics Education (NaCoME)*. IOP Publishing. doi:10.1088/1742-6596/1480/1/012026

Padmadewi, N. N. (2016). Techniques of Promoting Autonomous Learning in Classroom. *Journal of Education and Social Sciences, 3(*1), 45-52.

Saifullah, L. M., & Qohar, A. (2020). Development of SOP (Simulation of Proportion) Learning Media: Simulation Based Sticker Book on Direct and Inverse Proportion. *International Journal of Trends in Mathematics Education Research, 3(*2), 101-105.

Suhendri, H. (2011). Pengaruh Kecerdasan Matematis–Logis dan Kemandirian Belajar terhadap Hasil Belajar Matematika. *Jurnal Formatif: Jurnal, 1(*1), 29-39. (Translation: Suhendri, H. (2011). The Effect of Mathematical–Logical Intelligence and Learning Independence on Mathematics Learning Outcomes. *Formative Journals: Journals, 1(*1), 29-39).

Sumarto, S. N., Galen, F. v., Zulkardi, & Darmawijoyo. (2014). Proportional Reasoning: How do the 4th Graders Use Their Intuitive Understanding. *International Education Studies, 7*(1), *69-80.*

Citation of this Article:
Saleem, T., Mukhtar, S., & Aziz, S. (2021). Teaching direct and inverse proportion to 5th grade students through pictures and real-life problems, leads towards autonomous learning. *Journal of Science Education, 3*(1), 63-80.