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Analysis of Students' Errors in Geometry and Measurement in the Subject of Mathematics at Elementary Grades: A Study from the Mountainous Region of Pakistan

Pervaiz Alam*
Sadia Bhutta**
Sohail Ahmad***

Abstract

Geometry and Measurement (GM) concepts are one of the significant contents of mathematics that improves students' various skills including visualization, critical thinking, problem-solving, and deductive reasoning. More importantly, GM is not only connected with various other branches of mathematics, but it is also coupled with numerous other disciplines. It plays a crucial role in our daily activities and its importance cannot be overemphasized. Mathematics has significant place in school curriculum that is why the low achievement in mathematics has been focused throughout the world, and Pakistan with no exception. Among other reasons, errors in GM play a crucial role in this regard, and it needs to be thoroughly investigated. Therefore, this study aimed to analyze errors in GM of students (n=1006) enrolled in elementary grades (5, 6, and 8) in public and private schools of Gilgit-Baltistan. Students' responses to the items of GM were analyzed by identifying the most difficult items and discussing the types of errors with viable reasons. The results depict that the students of grades 5 and 6 found it very difficult to calculate the perimeter of a given rectangle and draw parallel line, respectively. Whereas grade 8 students found the measurement of angles in a triangle and calculation of the area of a 2D shape the most difficult items. The paper speculates reasons for these errors and proposes recommendations for the relevant stakeholders.

Keywords: Geometry and measurement, area and perimeter, angles in triangles, parallel and perpendicular lines

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* Pervaiz Alam is working as a research associate at Aga Khan University Institute for Educational Development (AKU-IED) Karachi Pakistan. He can be reached via Pervaz.alam2@aku.edu

** Sadia Bhutta is an associate professor at Aga Khan University Institute for Educational Development (AKU-IED) Karachi Pakistan.

*** Mr. Sohail Ahmad is PhD student at Aga Khan University Institute for Educational Development (AKU-IED) Karachi Pakistan. He can be reached via sohail.ahmad2@scholar.aku.edu

Introduction

Mathematics is considered essential as it has numerous practical usages in everyday tasks (Akhter & Usmani, 2018; Marchis, 2012; Mukhubele, 2012). It has become increasingly significant due to recent technological advancements, and it is now viewed as a crucial component in the progress of information and technology (Ozkan et al., 2018). Moreover, it is the root of science and technology; having said that, a significant number of pupils feel nervous and reluctant to learn mathematics (Adolphus, 2011; Zulnaidi & Oktavika, 2018) and many learners neglect math as their preferred domain of study (Furner & Duffy, 2002; Zulnaidi & Oktavika, 2018). Literature suggests numerous factors that contribute to this regard. However, the key factor is students' low achievement in tests and exams owing to misunderstandings and the errors they make in Mathematics. Low performance has been a matter of concern for the stakeholders not only in a national context but also in an international context.

Increasing a nation's production and competitiveness has historically depended heavily on its level of mathematical literacy. Better achievement in mathematics contributes in various ways, for instance, research by Eric Hanushek of Stanford University found a positive correlation between economic growth and student math achievement (Melkadze, 2012). Despite its importance numerous studies from the international context depict poor performance in mathematics (Mabena et al., 2021; Mbugua et al, 2012). Likewise, numerous extensive studies have been conducted in the national setting to appraise students' mathematics performance, and the results have indicated dismal performance. For instance, according to the Annual Status of Education Report (ASER, 2019), 22% of students in grade 3 and 57% of students in grade 5 can perform two-digit number division in Mathematics. Besides, Pakistan for the first time in 2019, took part in the Trend in International Mathematics and Science Study (TIMSS), and the outcomes demonstrated dismal proficiency in mathematics among grade 4 pupils (TIMSS, 2019). Among the 58 participating nations, Pakistan ranks next to last in the mathematics assessment (Akmal & Crawfurd, 2020; Halai, 2021; TIMSS, 2019).

The TIMSS findings revealed that grade 4 students' overall performance in mathematics was unsatisfactory, but curiously, a study of the data by content reveals that the students had particular difficulty with geometry (Halai, 2021; Mullis et al., 2020). This finding is in accord with earlier research studies conducted in Pakistan, where geometry is one of the content areas where students' achievement has stayed under par (Tayyaba, 2010; Akhter & Usmani, 2018; Bhutta & Rizvi, 2022).

Based on the literature, GM is a significant part of the class mathematics, thus having a grasp of it is essential. It assists students to perform well in other subject areas by enabling them to comprehend various mathematical content domains (Luneta, 2015; Wright, 2003). Regardless of the significance of this subject matter, students' unsatisfactory achievement has persisted as a cause for concern. There may be a variety of plausible reasons for pupils' poor performance and mistakes in this subject matter. These include improper and inadequate instruction (Salim, 2014); teachers' methods of instruction; absence of prior knowledge (Aziz & Kang, 2021) as well as gaps in students' comprehension of fundamental notions (Adolphus, 2011; Ali, 2011). Additionally, lacking accurate vocabulary in the classroom may have a detrimental effect on students' achievement and a better understanding of geometry (Atebe & Schafer, 2008; Luneta, 2015). Since errors in geometry lead to dismal performance therefore, various reserches have been conducted to explore errors in geometry and measurement in the international context (Riastuti et al., 2017; Ada & Kurtulus, 2010; Rushton, 2014). Contrary to this, in developing countries like Pakistan, little attention has been given to this domain, as a result, there is a dearth of indigenous literature.

As GM is among the poor performing course in several extensive research (Akhter & Usmani, 2018; Bhutta & Rizvi, 2022; TIMSS, 2019), thus, this study aims to analyze the data on students' tests from a perspective of exploring students' errors in GM. Knowing students' errors in GM might facilitate researchers, educators, and teachers to retain the possible errors in mind and the probable reason behind dealing with those in more constructive ways. Furthermore, understanding errors might aid in developing lesson plans that effectively differentiate between the interconnected concepts of GM, which students often confuse with one another. Besides, the findings might facilitate developing better teacher professional development programs by knowing the students' various errors in GM and equipping teachers to mitigate those errors.

Aim of the study

The study was designed to explore the students' errors in GM at primary and lower secondary schools in Gilgit-Baltistan using the existing data of a nationwide HEC-NRPU funded study.

Research Major Question

What Geometry and Measurement concepts do elementary school students struggle with most in the mountainous regions of Gilgit-Baltistan?

Literature Review

Geometry and Measurement an Essential Domain

Geometry and Measurement (GM) is a branch of mathematics that focuses on the examination of various flat or three-dimensional shapes. It is important in daily life since it encompasses a variety of ideas that people deal with in their daily lives and has many applications in a variety of professions (Akhter & Usmani, 2018; Quintero & Rosario, 2016). Other disciplines including Physics, Astronomy, Art, Mechanical drawing, Chemistry, Biology, Geology, and Technology require an excellent grasp of geometry to perform better (Aziz & Kang, 2021; Luneta, 2015). Learning GM also boosts the likelihood of doing better in other areas of mathematics due to the correlation between distinct GM principles and other ideas. Besides, making sure that kids are prepared to understand geometry is vital because GM problems may transform into number or algebra problems (Aziz & Kang, 2021; Wright, 2003). Learner may increase their visualization, critical thinking, perspective-taking, intuition, problem-solving, deductive reasoning, logical argument, and evidence skills by studying GM (Jones, 2002; Mukhubele, 2014). Additionally, mastering GM enhances students' spatial cognition, or their ability to precisely differentiate the visual world (Marchis, 2012; Quintero & Rosario, 2016). Changing mental representations can boost self-assurance and foster the development of an intuitive sense of spatial thinking (Jones, 2002). Similar to other types of thinking, spatial cognition, and visual descriptions are crucial to mathematical reasoning (Merrill et al., 2010; Aziz & Kang, 2021). It may be inferred that studying GM improves students' spatial ability, which enables them to comprehend the world more fully. Therefore, more emphasis should be given to learning GM at schools which consequently leads to better understanding and performance not only in GM but also in the other domains of mathematics.

Pupils' Achievement in GM

The academic performance of students, particularly in mathematics and specifically in the field of geometry and measurement, has been a cause for concern. Numerous studies on students' geometry achievement have found poor results in comparison to other Mathematics courses (Akhter & Usmani, 2018; Bhutta & Rizvi, 2022). Numerous variables, such as Lack of background information, level of mathematical proficiency, learning style, number of students in class, classroom organization, and teachers' methods of instruction, could contribute to students' inferior performance in GM (Aziz & Kang, 2021; Gloria, 2015). Aforesaid barriers contribute, either directly or indirectly, to the errors committed by students, which affect their achievement. Students struggle with geometry, which causes them to make errors in this subject (Biber et al., 2013; Kadaris et al., 2020; Luneta, 2015). According to the literature, pupils make a variety of mistakes when working with geometrical concepts. Students, for example, struggle to understand how different quadrilaterals and triangles are included in class based on their characteristics (Atebe & Schafer, 2008; Kadaris et al., 2020; Marches, 2012). The overgeneralization of angle conceptions may end up in a range of mistakes from students (Atebe & Schafer, 2008; Butoner & Filiz, 2017). As a result, pupils continue to make several mistakes while attempting to envision the perimeter and area of two-dimensional shapes. For example, students struggle to understand the idea of many rectangles with the same area but different perimeters (Machaba, 2016). Using the formulas for one notion for another is another illustration of confusion (Makonye, 2019). Literature also shows how difficult it is for pupils to comprehend the concept of "equidistance" when they draw parallel lines (Happs & Mansfield, 1992; Srinivas, 2019).

These errors prevent children from performing efficiently in mathematics. Regardless of the difficulty of the entire mathematics curriculum, GM is particularly challenging for pupils in schools (Akhter & Usmani, 2018; Sinclair & Bruce, 2015; Tayyaba, 2010). Exam failure has been pervasive as a result of the difficult nature of mathematics in general and GM in particular (Bhagat & Chang, 2015). Students may develop unfavorable attitudes toward learning mathematics as a result of this failure, and many may lose interest in the subject altogether.

Methodology

This study was funded by the Higher Education Commission (HEC) as a part of its National Research Program for Universities (HEC-NRPU) between 2018 to 2021. In that study, a mixed-method approach was employed to explore the link between elementary school teachers' pedagogy for teaching science and mathematics and their students' learning outcomes. However, scrutiny of the students' errors across different spheres of mathematics was not pertaining to the scope of the study. Thus, to analyze students' errors in GM the current study was carried out using secondary data on the performance of students from Gilgit-Baltistan (GB).

Sample

To investigate students' errors in GM, data on students' performance in Mathematics from Gilgit-Baltistan was obtained. The participants were recruited from two districts of GB - Gilgit (n=507; 50.4%) and Skardu (n=499; 49.6%) – having almost equal number of participant students. A similar pattern was observed while comparing the number of students across private (n=505; 50.2%) and public (n=501; 49.8%) school. However, gender comparison depicts a greater percentage of girls (n=548; 54.4%) in comparison to boys (n=458; 45.6%).

Structure of the Measurement Tool

For each grade (5, 6, and 8) in the parent study, three different mathematics achievement tests (MATs) were used to gauge the achievement of pupils across different topics of school Mathematics. Since the current focused on the investigation of students' errors in geometry and measurement, therefore, the MAT items for each grade level related to geometry and measurement notions were under consideration. The items distribution based on the content covered in MATs for each grade is shown in Table 1.

Table 1. *Distribution of Items as per Concepts*

Grade	**Concepts**				**Sum**
	Lines	Angles	Area and perimeter	Shapes	
5	1	0	3	2	6
6	1	0	4	1	6
8	1	4	1	2	8
Total	3	4	8	5	20

A total of 20 items from a variety of topics, including perimeter and area (n=08), shapes (n=5), lines (n=4), and angles (n=03) were used in MATs to assess student's achievement in GM.

Analytical procedure

The data were analyzed using SPSS 23. The Difficulty Index (DI) of each question was computed for each grade 5, 6, and 8 to investigate the geometric errors. The percentage of participants who answered a test question accurately is known as the DI (Sharma, 2021). Additionally, DI enabled us to pinpoint the geometry notions pupils struggled with most frequently. As noted in Table 2, DI has complexity levels ranging from 0.00 to 1.00.

Table 2. *Criteria for Difficulty Index*

S#	DI range	Category
1	Less than 0.20	Most difficult
2	0.21-0.40	Difficult
3	0.41-0.60	Moderately difficult
4	0.61-0.80	Moderately easy
5	0.81-1.00	Easy

The criteria that were established for finding the most challenging items for this study stated that items with a DI less than 0.20 would be the most challenging (Hingorjo & Jaleel, 2012; Sharma, 2021). This approach assisted in categorizing challenging notions so that patterns of errors in specific items could be further investigated.

Results

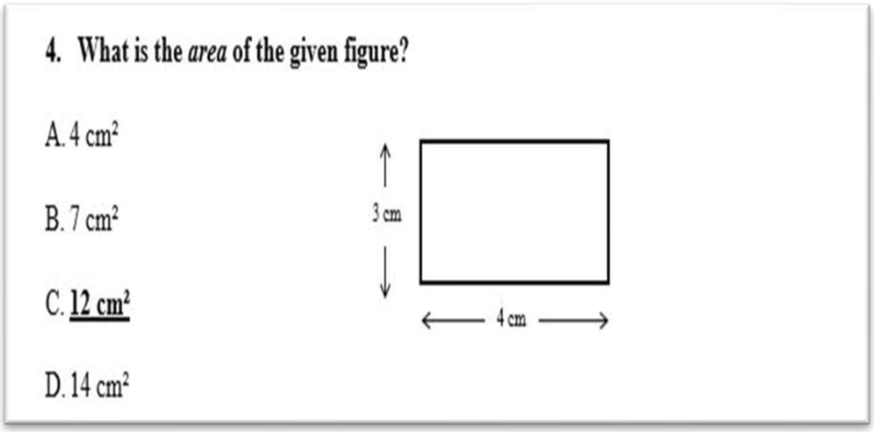
The degree of difficulty of the GM items was determined using the Difficulty Index (DI) since the study's objective was to examine frequently recurring errors committed by the pupils. The items that had a DI less than or equal to 0.20 were deemed to be the most challenging, and they were looked at more closely to find out the nature of the errors and their possible causes. Out of the 20 items, 05 were classified as being the most difficult, including the same number of items (i.e., 01) from grades 5 and 6, while at the same time 03 items from grade 8. As revealed in Table 3, this category has a total of 04 multiple-choice questions (MCQs) and 1 constructed response question (CRQ).

Table 3. *The Most Challenging Items*

Grade	***Notions***	***Accurate responses n (%)***	***Inaccurate responses n (%)***	***Unattempt n (%)***	***Item type***
5	Computing the area of a rectangle	37 (12.0)	283 (85.0)	11 (3.0)	MCQ
6	Construction of Parallel Line Next to given Line	65 (18.0)	301 (81.7)	1 (0.3)	CRQ
8	Right-Angled Triangle with a Missing Angle	62 (20.0)	241 (78.0)	5 (2.0)	MCQ
	Calculating the Area of Composite 2D Shapes	56 (18.0)	242 (78.0)	10 (4.0)	MCQ
	Missing Angles in Triangles	37 (12.0)	269 (87.4)	2 (0.6)	MCQ

Grade 5: Computing the Area of Rectangle. Grade 5 students discovered a question related to area and perimeter the most difficult. Figure 1 depicts the item along with stem, distractors, and key.

Figure 1
Computing the Area of Rectangle



This application-driven question was used to gauge how well students could use their understanding of area calculation. The right answer in this situation relies on the student's familiarity with the supplied shape (a square, rectangle, or other 2D polygons), as well as their understanding of the pertinent formula. Table 4 demonstrates the students' answers.

Table 4. *The selected options*

Options	**No. of participants n (%)**
A	83 (25%)
B	149 (45%)
C*	37 (12%)
D	51 (15%)

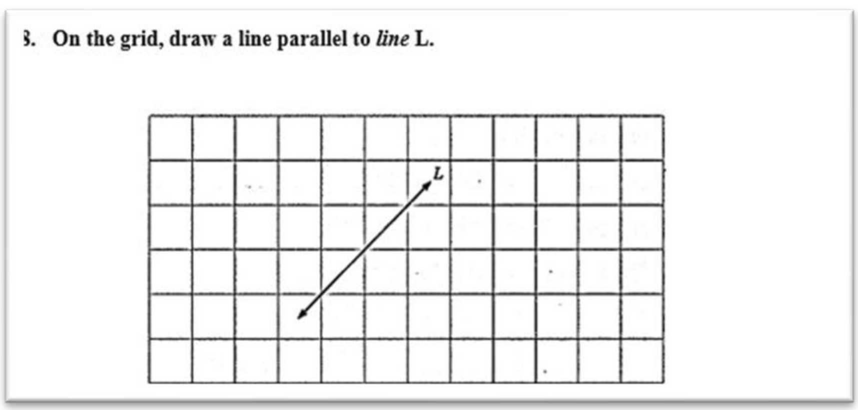
The findings show an astounding majority (n=283; 85%) of pupils chose the wrong answers. Curiously, option 'B' (i.e.,$7cm^2$) proved to be the student preference since less than half (n=149; 45%) selected it. This shows that even without considering the unit (i.e., 4 cm + 3 cm), pupils thought of the semi-perimeter as an area of the given rectangle. On the other hand, 25% (n=83) of the respondents chose the option "A" ($4cm^2$).

This demonstrates how learners might consider the area to be the rectangle's longest side. Similar to this, a portion of participants (n=51; 15%) selected the distractor "D" (i.e., $14cm^2$). The choice of this diversion suggests that some students may have amalgamated the ideas of perimeter and area without even considering the SI units.

Although the fundamental idea behind this question is covered in the primary school mathematics curriculum, it prompts the question of, why so numerous individuals choose the wrong answers. The inclusion of these two ideas (area and perimeter) in Pakistani textbooks may be one of the justifications. The textbooks provide area and perimeter concepts concurrently, and instructors also taught students in the same order. Because of this, it may have been challenging for students to discern between the two ideas, which may have led them to combine them when applying their knowledge in a practical setting.

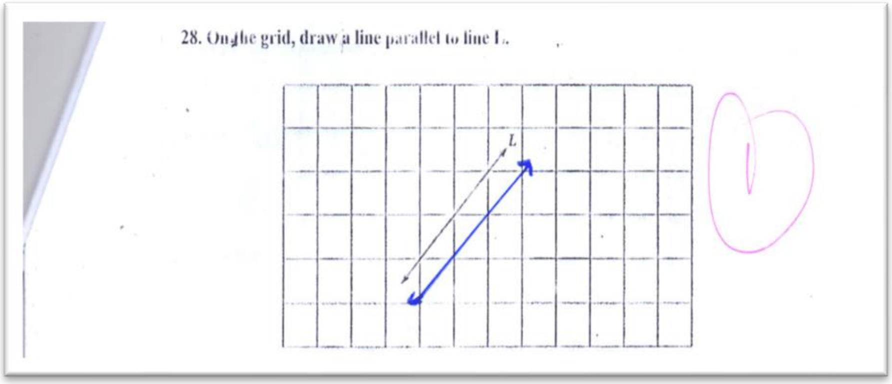
Grade 6: Construction of a Parallel. In this question, students were asked to construct a parallel line on the grid paper adjacent to the given line 'L'. The drawing of parallel lines was related to the constructive response question (CRQ) and was found to be one of the hardest items to resolve. In Figure 3, the CRQ is depicted.

Figure 3
Construction of Parallel Line



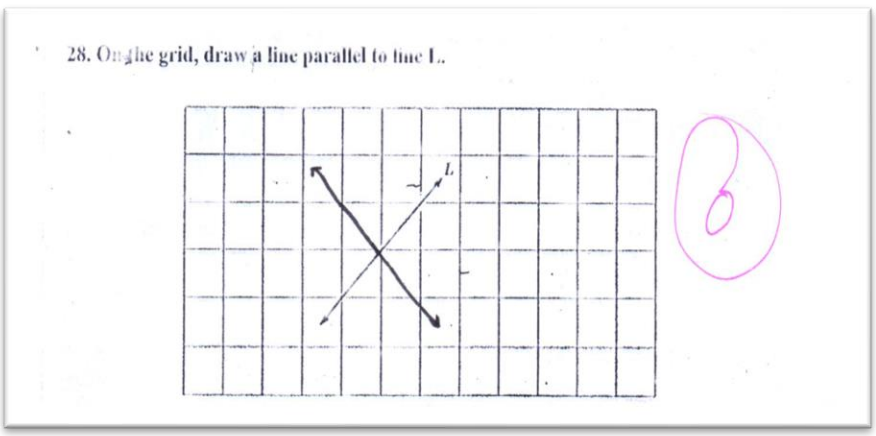
The performance of students in this task significantly relies on their familiarity with the definition of parallel lines, which is the term used to describe lines that are always spaced apart consistently and never cross one another. A parallel line adjacent to the given line "L" was correctly drawn by nearly one-fifth of the students (n=65; 18%). An illustration of the right responses from pupils is shown in Figure 4.

Figure 4
The Correct Response



Astonishingly, a large proportion (n=301; 81.7%) of students attempted it wrong. An illustration of the wrong answer is shown in Figure 5.

Figure 5
Inappropriate Response

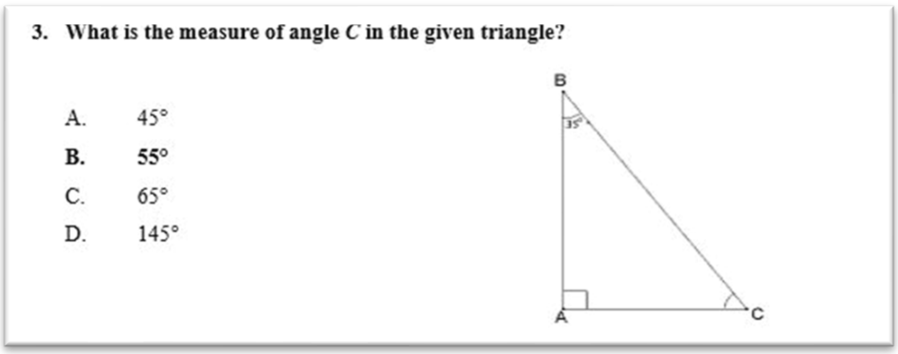


Scrutiny of the example in Figure 5 and related cases from the data facilitated the identification of some of the probable causes of the students' misperception regarding the subject addressed by this item. The major definition of a parallel line, which stresses only that "line should always be an equal distance apart," was unknown to pupils. Additionally, they may have combined the idea of a parallel line with a perpendicular line, which is an entirely distinct concept.

One of the viable explanations might be the inclusion of the concept in textbooks used throughout the GB. Transversal, parallel, and perpendicular line notions are presented simultaneously in the textbooks, and instructors similarly present these ideas without giving a range of episodes that would make it easier to distinguish amongst the concepts. Students as a result confuse the key differences between each of these domains.

Grade 8: Right-Angled Triangle with a Missing Angle. The purpose of this application-based question was to assess the capacity of learners to use their knowledge to compute the missing angle (i.e., ∠C). The correct and the incorrect answers are revealed in Figure 6.

Figure 6
Missing Angle in a Right-Angled Triangle



Recognizing the symbol for a right-angle (i.e., ∟= 90º) and having enough understanding of the triangle's sum angle properties (i.e., ∠A+∠B+∠C=180º) are prerequisites for figuring out the omitted angles in a right-angle triangle △ABC. The number of students who selected the various options as per their comprehension is shown in Table 5.

Table 5
The Selected options

Options	**No. of participants n (%)**
A	113 (37%)
B*	62 (20%)
C	71 (23%)
D	57 (19%)

The findings indicate a large proportion of students (n=241; 79%) preferred distractor over the right answer. The first option, (i.e., A=45º) was discovered to be the most frequently chosen wrong response among the three distractions since it had been chosen by 37% (n=113) of the students. The option suggests that pupils may recognize the right-angle sign. Nevertheless, they appeared to have missed the angle ‘∠B’, which is indicated in the figure as (i.e., ∠B=35^{o}). " Angles "B" and "C" would have been perceived as equal angles as a result (i.e., ∠B=∠C =45^{o}). Additionally, nearly one in five students (n=57; 19%) selected option “D” (i.e., 145^{0}). This choice shows that they intended to add angles ∠A and ∠B, but instead accidentally added the needed angle (i.e., ∠C=55^{o}). As a result, they had chosen 145^{o} (i.e., 90^{o}+55^{o}= 145^{o}) as the correct response. Further this implies that difficulties are encountered by pupils in identifying the known and required angles in a right-angle triangle. Whereas just above one-fifth of students (n=71; 23%) selected option “C” (65^{o}). In conclusion, students struggle to determine the missing angle in a given triangle using the basic angle sum property of a triangle.

The idea of missing angles is covered in schoolbooks used in both public and private schools throughout GB, including those published by Oxford University Press and the Punjab Textbook Board. Additionally, there are a sizable number of explained samples as well as problems in the exercise section of these books. But Students' performance fell short of expectations. Without considering the geometrical properties, students have merely concentrated on the visual aspects of triangle shapes. This necessitates a variety of instructional strategies to convey these fundamental ideas, which may cause difficulties for subsequent classes.

Grade 8: Calculating the Area of Composite 2D Shapes. This test item evaluated students' understanding of the procedure used to calculate the area of composite two-dimensional (2D) shapes. The item is shown in Figure 7 along with three distractors and an answer.

Figure 7

Calculating the Area of Composite 2D Shapes

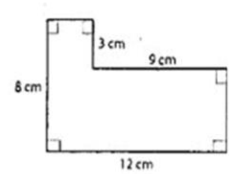
4. What is the area, in square cm, of the figure?

A. 66

B. 69

C. 81

D. 96



The fundamental idea needed to determine the area of composite shapes primarily relies on splitting a given shape into two or more shapes, each of which areas can be calculated separately and added later. The percentage of grade 8 students answers is shown in Table 6.

Table 6. *The Selected options*

Options	**No. of Pupils n (%)**
A	63 (20%)
B*	56 (18%)
C	84 (27%)
D	95 (31%)

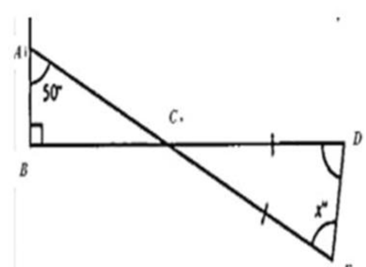
The findings show that the vast majority of students (n=242; 78%) chose incorrect the distractor. Approximately 1/3 (n=95; 31%) of participants chose option "D" (i.e., 96cm) as their preferred answer. The choice of this distractor shows that students estimated the area of the supplied composite shape by considering its length (L=12cm) and breadth (B=8cm), which led to selecting this alternative (i.e., $96cm^2$=12cm×8cm), without even breaking it down into two or more shapes. Similar to this, 20% (n=63) of the students chose option "A" ($66cm^2$) as the right response. This calculation indicates that they were able to determine the size of the rectangle ($60cm^2$), but they did not compute the area of a square; instead, they simply added the square's semi-perimeter (3 cm + 3 cm = 6 cm). Decomposing the composite shape into 2D shapes and estimating the area using formulae remained a challenging task for students.

Grade 8: Missing Angles in Triangles. This item was associated with estimating missing angles in triangles as represented in figure 8.

Figure 8

6. In the diagram, CD = CE. What is the value of *x*?

A. 40
B. 50
C. 60
D. 70



Estimating the missing Angles in Triangles

In this instance, choosing the right response is heavily reliant on (i) identifying the right-angle triangle and then applying the angle sum property followed by; (ii) identifying the vertically opposite angles (∠ACB=∠DCE); and (iii) understanding the fundamental property of isosceles triangles (i.e., that the two angles that face the two equal sides are equal). Based on their knowledge, the students chose numerous options which are shown in Table 7.

Table 7
The Selected Options

Options	**No. of participants n (%)**
A	62 (20%)
B	114 (37%)
C	93 (30%)
D*	37 (12%)

The findings show an enormous number (n=269; 87%) of students chose the wrong answer for this item. Additionally, option "B" (i.e., 50^o) continued to be the most prevalent response with just over one-third (n=114; 37%) picked it. The fact that the majority of students chose this option suggests that they may have thought about the angle $\angle BAC = \angle DEC$. They may have chosen the incorrect solution because they were thinking of the line $mAC = mCE$, that is totally improper comprehension. While less than 1/3 of students (n=93; 30%) chose option “C” (i.e., 60^o). The most probable explanation could be two little straight lines that stand in for an isosceles triangle's equal sides. It is very possible that students thought of the third side as being equal to the other two sides, which may have caused them to think of it as an equilateral triangle with ∠60° angles on each side. On the other hand, 20% (n=62) of the participants chose the option “A” (i.e., 40^0). In short, students have trouble remembering the fundamental characteristics of triangles. Why were so many students unable to choose the right answer although the relevant items had been repeated and were of great importance in lower secondary grades? One of the causes may be that learners only pay attention to the physical characteristics of geometric shapes and broad mathematical principles. This might have led students to choose various distractors.

Discussion

The findings revealed that students commit widespread errors like mixing the concepts of perimeter and area, and parallel lines with perpendicular lines. Besides, students are unable to calculate the missing angles in triangles. This segment delineates the results considering the literature. Major findings include:

Amalgamating the Notion of Perimeter and Area.

The notion of perimeter and area is prominent among students only in terms of knowing the formulae, and both concepts are difficult for students not only at the primary but also at the secondary level (Makonye, 2019). Mixing up the formulae for calculating the perimeter and area is a common error made by students in geometry. The formula for calculating the perimeter is the sum of the lengths of all sides of a two-dimensional shape, while the formula for calculating the area is the measure of the region enclosed by the shape. These two formulae are distinct and mixing them up can lead to incorrect answers. To avoid such errors, students need to have a clear understanding of the difference between the two formulae and the situations in which each formula is applied. This mixture is highlighted in this study in line with a quasi-experimental research study revealing that students were misusing one formula for the other (Makonye, 2019). Besides, the findings also accord with the study, where the research has found inadequate prior knowledge pertaining to area and perimeter cases misunderstanding about area and perimeter (Machaba, 2016). To help students better understand and differentiate between two related but distinct concepts, teachers may need to explicitly highlight the differences and similarities between them. This could involve teaching one concept and its application, followed by teaching the other concept and its application and then comparing the two concepts to help students see how they are related but different. By doing so, students may be able to develop a more nuanced and deeper understanding of each concept, which could facilitate their ability to apply their knowledge in various situations.

Incorrect use of the Angle Sum Properties of Triangles

In any right-angled triangle, the sign of a right angle (∟) aids to recognize the triangle as a right-angled triangle and further, this recommends that the summation of the other two angles will be 90 degrees. However, this study found that a substantial number of pupils lacked an understanding of the angle sum property. This could contribute to the widespread use of rote learning as the primary pedagogical strategy for teaching mathematics in the country, which may result in limited conceptual understanding among students (Bhutta & Rizvi, 2022). Subsequently, Students may not internalize mathematical concepts in a way that allows them to apply their knowledge effectively in various situations, whether familiar or unfamiliar. This suggests that their understanding of the concepts is not deep enough to allow for meaningful application and problem-solving.

Perpendicular or Parallel line

Comprehending parallel lines is crucial to comprehend a range of other concepts, such as the categorization of polygons, the relationships between angles, and geometric proofs. Students' knowledge of the notion is strongly dependent on their acquaintance with the idea of "equidistance," which is an important part of how most students perceive parallel lines in general (Srinivas, 2019; Mansfield & Happs, 1992). The study's findings revealed that a sizable number of students lacked familiarity with the fundamental concept of equidistance in parallel lines, leading to a common error where students confused parallel lines with perpendicular lines. However, concepts such as parallel lines are not confined to the classroom and can be introduced through visual exposure and real-life examples. Students who engage in visual challenges and activities tend to develop stronger visualization skills and problem-solving abilities (Shirali, 2020). As a result, there is an urgent need to raise students' visualizing abilities, which may assist them to do better not only in GM but also in other domains of Mathematics.

Conclusion

In a nutshell, learners encounter innovative ideas as they build their knowledge, update their previous understanding, and align with updated knowledge, which can lead to errors. In mathematics, each class builds on the information learned in the previous class, and errors are a natural part of the learning process. However, if these errors go unnoticed, they can persist and even worsen over time. Timely identification and addressing of errors can help improve the foundation and enhance learning in the future. This study found that students tend to make numerous errors in various aspects of geometry, particularly in high order thinking tasks that require application and reasoning in both familiar and unfamiliar situations. The study's results may also suggest that mathematics education places more emphasis on transmitting knowledge rather than fostering students' conceptual understanding (Rind & Mughal, 2020). As a result, this reinforces the traditional view of learners as passive recipients of knowledge and teachers as the sole source of information. To promote conceptual understanding and problem-solving abilities, it is essential to bridge the gap between informal and formal mathematics, as students engage in mathematical thinking in their daily lives.

Recommendations

For Teaching: The study found that students made numerous errors in the domain of geometry and measurement (i.e., perimeter and area, missing angles, and parallel lines). These results can support teachers to anticipate the common errors that students make and prepare lessons that address the underlying concepts contributing to these errors. Since geometry is indispensable to understanding other subjects, this information could also be useful for teacher education institutes to include in their curriculum, helping teachers to develop effective teaching strategies for teaching and geometry.

For Empirical studies: This current research study provides instrumental insights into students' errors in geometrical concepts. The findings call for intervention-based studies using latest technologies to mitigate students' errors and improve students' learning outcomes.

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